Exercise 3.2.1: Consider a relation with schema R(A,B,C ,D ) and FD’s AB -> C , C -> D , and D -> A.  
L={B}, R={0}, M={ACD}

{B}+={B}

{AB}+={ABCD} - key

{AC}+={ACD}

{AD}+={AD}

{BC}+={BCDA} - key

{BD}+={BDAC} - key

{ABC}+={ABD}+={BCD}+ ={BCDA}

{ACD}+={ACD}

1. Non-trivial FD:  
   C→ A , AB→ D , AC→D , BC→A , BC→D , BD→A , BD→C , CD→A , ABC→D , ABD→C , BCD→A
2. Key : AB, BC , BD
3. Super key that are not keys : ABC , ABD , BCD ,ABCD

Exercise 3.2.2: Repeat Exercise 3.2.1 for the following schemas and sets of FD’s:  
A, S(A, B, C, D) with FD’s A → B, B → C, and B → D.

{A}+ = {ABCD} - key {BC}+ = {BCD} {ABCD} - super key

{B}+ = {BCD} {BD}+ = {BDC}

{C}+ = {C} {CD}+ = {CD}

{D}+ = {D} {ABC}+ = {ABCD} - super key

{AB}+ = {ABCD} - key {ABD}+ = {ABDC} - super key

{AC}+ = {ACBD} - key {BCD}+ = {BCD}

{AD}+ = {ADBC} - key {ACD}+ = {ACDB} - super key

B, T(A, B, C, D) with FD’s AB → C, BC → D, CD →A, and AD → B.

{A}+ = {A} {AC}+ = {AC} {ABD}+ = {ABDC} - super key

{B}+ = {B} {AD}+ = {ADBC} - key {BCD}+ = {BCDA} - super key

{C}+ = {C} {BC}+ = {BCDA} - key {ACD}+ = {ADCB} - super key

{D}+ = {D} {AB}+ = {ABCD} - key {ABCD} - super key

{CD}+ = {CDAB} - key {ABC}+ = {ABCD} - super key

C, U(A, B, C, D) with FD’s A →B, B → C, C → D, and D → A.

{A}+ = {ABCD} {AC}+ = {ACBD} - key {ABC}+ = {ABCD} - super key  
{B}+ = {BCD} {AD}+ = {ADBC} - key {ABD}+ = {ABDC} - super key  
{C}+ = {CDAB} {BC}+ = {BCDA} - key {BCD}+ = {BCDA} - super key  
{D}+ = {DABC} {BD}+ = {BDCA} - key {ACD}+ = {ACDB} - super key  
{AB}+ = {ABCD}- key {CD}+ = {CDAB} - key {ABCD} - super key

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Exercise 3.2.9: Find all the minimal bases for the FD’s and relation of Example 3.11

{A→B, B→A, B→C, C→B} {C→ A, A→ B, B→ A, B→ C}

{A→B, B→C, C→A} {C→ B, C→ A, B→ C, A→ B}

{B→A, A→C, C→A} {B→ C, B→ A, A→ B, C→ B}

{C→A, A→B, B→C} {A→ B, A→ C, B→ A, C→ B}

{A→C, B→A, C→A, C→B}

Exercise 3.2.10: Suppose we have relation R(A,B,C,D ,E), with some set of FD’s, and we wish to project those FD’s onto relation S(A, B, C). Give the FD’s that hold in S if the FD’s for R are:

1. AB →DE , C→E , D →C and E → A  
   {AB}+ = {ABDEC} so AB🡪C holds in S  
   {C}+ = {CEA} so C🡪A holds in S
2. A→D, BD→E, AC→E, DE→B  
   {A}+ = {AD} so A→B holds in S  
   {AC}+ = {ACE} so A→B, C→B holds in S
3. AB→D, AC→E, BC→D, D→A, E→B  
   {AB}+ = {ABD} so B→A, A→B holds in S  
   {AC}+ = {ACEBD} and {BC}+ = {BCDAE}   
   So A→B, B→A, C→B, C→A holds in S
4. A→B, B→C, C→D, D→E, E→A  
   {A}+ = {A}  
   {B}+ = {BCDEA}  
   {C}+ = {CDEAB}  
   So A→B, B→C, C→A holds in S